Strengthening the core in Calculus: Differentiation and Integration

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Introduction

Calculus plays a vital role in mathematics and is the important branch in Applied Mathematics. Calculus covers from limit, functions, derivatives, and integral till infinite series. Understanding key concepts in Algebra and Trigonometry is needed to have a better comprehension in Calculus.

As Calculus is widely applied in the field of engineering, sciences and economics, it is essential for students to learn this subject in schools and higher education institutes. Nevertheless, students struggle and perceive Calculus as a hard and difficult subject where they often misunderstood the concept of calculus (Tarmizi 2010).

Students generally have difficulties in solving problems with limits, derivative and integral (Siti Fatimah, 2019). Therefore, this article serves as a tool for students to grasp the heart of calculus which consists of differentiation and integration. It also tackles on basic algebra and trigonometry, definitions as well as techniques of differentiation and integration.

Tips in Learning Calculus

Remember the Basics Mathematics

It is important to memorize the basics in mathematics to avoid silly mistakes in the solutions. There is no shortcut as these basic properties are being used in all of Calculus's branches. Figure 1 below shows a right triangle where the adjacent side is labelled by a, the opposite side is labelled by b and side c is the hypotenuse of the triangle.

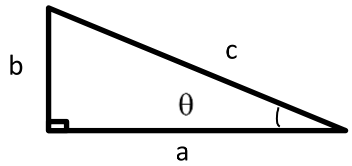

Figure 1

The angle θ in Figure 1, can be calculated based on the trigonometric rules as follows,

$$\sin\theta = \frac{b}{c},\ \cos\theta = \frac{a}{c},\ \tan\theta = \frac{b}{a},\ \csc\theta = \frac{c}{b},\ \sec\theta = \frac{c}{a},\ \cot\theta = \frac{a}{b}$$

Table 1 and Table 2 below show the law of indices and the laws of limits respectively that are widely used in calculus.

Table 1: Laws of Indices

(1) $a^m a^n = a^{m+n}$	(5) $\left(\frac{a}{b}\right)^n = \left(\frac{a^n}{b^n}\right), b \neq 0$	(8) $a^{-m} = \frac{1}{a^m}$
(2) $\frac{a^m}{a^n} = a^{m-n},\ a \neq 0$	(6) $a^0 = 1$	(9) $a^{\frac{1}{m}} = \sqrt[m]{a}$
(3) $\left(a^m\right)^n = a^{mn}$	(7) $a^1 = a$	(10) $a^{\frac{n}{m}} = \sqrt[m]{a^n} = \left(\sqrt[m]{a}\right)^n$
(4) $(ab)^n = a^n b^n$		

Table 2: Laws of Limits

Let k and a be any real number,		
(1) $\lim\limits_{x\to a} k = k$	(4) $\lim\limits_{x\to 0^+} \frac{1}{x} = \infty$	(7) $\lim\limits_{x\to +\infty} x = +\infty$
(2) $\lim\limits_{x\to a} x = a$	(5) $\lim\limits_{x\to \pm\infty} k = k$	(8) $\lim\limits_{x\to \pm\infty} kf(x) = k \lim\limits_{x\to \pm\infty} f(x)$
(3) $\lim\limits_{x\to 0^-} \frac{1}{x} = -\infty$	(6) $\lim\limits_{x\to -\infty} x = -\infty$	

Understand the Definition

It is vital to understand the definitions in limits, derivatives and integrals. All of them are the foundations and core in Calculus. When students fully understand the definitions and concepts, no matter how the questions are being rephrased, they will surely know how to solve the questions in their hands.

Rules in Mathematics

There are always rules in all fields of study including Calculus. In differentiation, there are product rule, quotient rule as well as chain rule. Integration has a lot more rules such as integration by substitution and integration by parts.

Understand the Problem

The problems given may be a direct or indirect where critical thinking is often needed to solve ones. By doing lots of exercises, it can help students to quickly understand the problem in hand and hence manage to solve them correctly.

Differentiation

Definition 1

The derivative of the function f is the function f' given by $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$. The process of computing a derivative is called differentiation. If $y = f(x)$, then $y' = \frac{dy}{dx} = \frac{d}{dx} f(x)$. The expression $\frac{d}{dx}$ is called a differential operator. Table 3 below shows a basic rules to compute derivatives:

Table 3: The Derivatives of a Function

(1) Constant Rule: $\frac{d}{dx}(k) = 0$	(5) Trigonometric Rules: $\frac{d}{dx}\sin\left[f(x)\right] = f'(x)\cdot\cos\left[f(x)\right]$
(2) Power Rule: $\frac{d}{dx}(x^n) = nx^{n-1}$	$\frac{d}{dx}\cos\left[f(x)\right] = f'(x)\cdot\left(-\sin\left[f(x)\right]\right)$
	$\frac{d}{dx}\tan\left[f(x)\right] = f'(x)\cdot\sec^2\left[f(x)\right]$
(3) Logarithmic Rule: $\frac{d}{dx}\ln\left[f(x)\right] = \frac{f'(x)}{f(x)}$	$\frac{d}{dx}\mathrm{cosec}\left[f(x)\right] = f'(x)\cdot\left(-\mathrm{cosec}\left[f(x)\right]\cot\left[f(x)\right]\right)$
(4) Exponential Rule: $\frac{d}{dx}e^{f(x)} = f'(x)\,e^{f(x)}$	$\frac{d}{dx}\sec\left[f(x)\right] = f'(x)\cdot\left(\sec\left[f(x)\right]\tan\left[f(x)\right]\right)$
	$\frac{d}{dx}\cot\left[f(x)\right] = f'(x)\cdot\left(-\mathrm{cosec}^2\left[f(x)\right]\right)$

If given two functions such as $f(x)$ and $g(x)$, the derivatives can be computed as below.

Product Rule

Suppose that f and g are differentiable. Then, $\frac{d}{dx}[f(x)\,g(x)] = f'(x)\,g(x) + f(x)\,g'(x)$

or let $u = f(x)$ and $v = g(x)$, then $\frac{dy}{dx} = uv' + vu'$.

Quotient Rule

Suppose that f and g are differentiable. Then, $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{\left[g(x)\right]^2}$

or let $u = f(x)$ and $v = g(x)$, then $\frac{dy}{dx} = \frac{vu' - uv'}{v^2}$.

Integration

Definition 2

The process of finding anti-derivatives is called anti-differentiation or integration. If $\frac{d}{dx}[F(x)] = f(x)$, then, integrating $f(x)$ produces the anti-derivatives $F(x) + C$ where *C* is a constant. It also can be denoted as $\int f(x)\,dx = F(x) + C$. Table 4 shows the list of the basic integration formula.

Table 4: List of the Basic Integration formula

(1) Constant Rule:	(5) Trigonometric Rules:
$\int k\,dx = kx + C$	$\int \sin x\,dx = -\cos x + C$
(2) Power Rule:	$\int \cos x\,dx = \sin x + C$
$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad \int \frac{1}{x} dx = \ln\lvert x\rvert + C$	$\int \tan x\,dx = -\ln\lvert\cos x\rvert + C$
(3) Logarithmic Rule:	$\int \cot x\,dx = \ln\lvert\sin x\rvert + C$
$\int \ln\lvert x\rvert\,du = x\ln\lvert x\rvert - x + C$	$\int \sec x\,dx = \ln\lvert\sec x + \tan x\rvert + C$
(4) Exponential Rule:	$\int \sec^2 x\,dx = \tan x + C$
$\int e^x\,dx = e^x + C$	$\int \mathrm{cosec}^2\,x\,dx = -\cot x + C$

There are several methods of integration that can be used to solve more complicated integral.

Integration by substitution

The integration by substitution method also called as u-substitution is used when an integral contains some function and its derivative. In this case, u will be set as equal to the function and rewrite the integral in terms of the new variable u. This makes the integral easier to solve. Then, the final answer will be expressed in terms of the original variable x.

Integration by parts

This method is applied when integrating the product of two different functions of the same variable. The formula is $\int u\,dv = uv - \int v\,du$. In order to solve the integral using integration by parts, the integral is split into two parts, *u* and *dv*, where *u* will be differentiated and *dv* will be integrated. There is a guideline to select *u* and *dv* that is by using acronym L-I-A-T-E where L stands for logarithmic function, I stands for inverse trigonometric function, A stands for algebraic function, T stands for trigonometric function, and E stands for exponential function. According to this guideline, *u* should be the function that comes first in this list and dv will be the rest of the function.

Integration by trigonometric substitution

Integration by trigonometric substitution method is used to solve the integrals containing the expression of the form $\sqrt{a^2 - x^2}$, $\sqrt{x^2 - a^2}$ and $\sqrt{a^2 + x^2}$ .This method uses substitution to rewrite these integrals as trigonometric integrals where the variable x changed to θ by the substitution, then the identity allows to solve of the root sign. Table 5 shows the list of trigonometric substitutions for the given radical expressions.

Table 5: List of Trigonometric Substitution

Expression	**Substitution**	**Identity**
$\sqrt{a^2 - x^2}$	$x = a\sin\theta$	$1 - \sin^2\theta = \cos^2\theta$
$\sqrt{x^2 - a^2}$	$x = a\sec\theta$	$\sec^2\theta - 1 = \tan^2\theta$
$\sqrt{a^2 + x^2}$	$x = a\tan\theta$	$1 + \tan^2\theta = \sec^2\theta$

Integration by partial fractions

Partial fraction is a technique of integration that can be used to integrate a proper rational function. A ratio of two polynomials $\frac{P(x)}{Q(x)}$ is called as a rational function where it is said to be proper when the degree of $P(x)$ is lesser than degree of $Q(x)$. This method focuses on denominators that can be factorized into linear and quadratic equations where the integral can be written as a sum of simpler rational functions so that the integral can be solved easily. Table 6 shows some simple partial fractions.

Table 6: Form of Partial Fraction

	Form of the rational function	Form of the partial fraction
1	$\frac{px+q}{(x-a)(x-b)}, a \neq b$	$\frac{A}{x-a}+\frac{B}{x-b}$
2	$\frac{px+q}{(x-a)^2}$	$\frac{A}{x-a}+\frac{B}{(x-a)^2}$
3	$\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$	$\frac{A}{x-a}+\frac{Bx+C}{x^2+bx+c}$
4	$\frac{px^2+qx+r}{(x^2+bx+c)^2}$	$\frac{Ax+B}{x^2+bx+c}+\frac{Cx+D}{(x^2+bx+c)^2}$

where x^2+bx+c cannot be factorized further

Conclusion

In conclusion, the main problem that needs to tackle in Calculus is students' comprehension in differentiation and integration and more importantly in algebra and trigonometry. Teaching problem-solving strategies in Calculus is necessary so that students know the steps toward solution formulation. Memorizing the basic Mathematics, grasping the definition and concept will then help students to identify which rules or methods needed to solve problems given to them. Students also need to do more practice to improve their ability in solving application problems (Klymchuk,2010).

References:

Tarmizi, Rohani Ahmad. 2010. "Visualizing Students' Difficulties in Learning Calculus." *Procedia - Social and Behavioral Sciences* 8: 377–83.

Siti Fatimah, Yerizon. 2019. "Analysis of Difficulty Learning Calculus Subject for Mathematical Education Students." *International Journal of Scientific and Technology Research* 8: 3: 80-84.

Klymchuk, Sergiy & Zverkova, Tatyana & Gruenwald, Norbert & Sauerbier, Gabriele. 2010. "University students' difficulties in solving application problems in calculus: Student perspectives." *Mathematics Education Research Journal* 22: 81-91.